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A FRACTAL BASED TECHNIQUE FOR IMAGE MAGNIFICATION

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A Fractal Based Technique For Image Magnification

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Abstract

A new method for image magnification using fractal based technique is proposed. We call this new technique fractal image magnification. The technique is designed assuming the self transformability property of images. The technique described here also utilizes Genetic Algorithm with elitist model that greatly decreases the search for finding the self similarities in the given image. The article presents theory and implementation of the proposed method. A simple distortion measure scheme is also proposed to judge the image quality of the magnified image. Comparison with one of the most popular magnification techniques, the nearest neighbor technique, is made.

Keywords: Image Magnification, Iterated Function System (IFS), Genetic Algorithm (GA).

1 Introduction

Image magnification is an important task which is performed to bring data from different sources and scales to a common scale. This type of preprocessing is very common in satellite image analysis [1, 2]. Moreover the image regions which are not explicitly visible in the original may provide more information after magnification. Like other image processing tasks, the magnification is performed on an input image which is represented as an array of pixel values and this requires huge memory for storage. In case of transmission of some coded version of the image, magnification can be performed at the receiving end only after reconstructing the image from its code. Thus at the receiving end, besides the storage problem, one needs to perform an extra computing task of decoding. So it is beneficial if one can reduce the storage requirements of an image or channel bandwidth in case of transmission of an image as well as some reduction in computing load. The formidable problem of huge memory requirements during magnification can greatly be minimized by developing a suitable magnification technique which will use the coded representation of the image instead of using the original image (uncoded image). Similarly in the case of transmission, if the coded version of the image is directly magnified at the receiving end without reconstruction then this will also reduce the cost of decoding at the receiving end. With this problem in mind an attempt is made to propose a new magnification technique which can be applied directly on the coded version of the image.

Fractal image coding technique is one of the efficient approximate image coding techniques available. In this technique utmost care is taken so that a reconstructed image is produced, from the coded version, which is subjectively very close to the original uncoded image. Actually the coded image representation must implicitly carry all the spatial information associated with the image. Besides the spatial information, the fractal codes carry the information of the self similarities present in the image. This self similarity property is also exploited in the proposed image magnification technique. We call it as fractal image magnification technique.

Fractal geometry has recently come into the limelight due to its use in various scientific and technological applications, specially in the field of computer based image processing. It is being successfully used for image data representation [3, 4] and as image processing tool [5, 6]. In this connection, the use of Iterative Function System (IFS) and Collage theorem [3] have shown a remarkable improvement in the quality of processing compared to that obtained using existing image processing techniques.

A fully automated fractal image compression scheme of digital images was first proposed by Jacquin [7]. The basic idea of fractal image compression or to find the fractal codes of an image is to approximate small blocks, called range blocks, of the given image from large blocks, called domain blocks, of the same image. Thus to find the fractal codes for a given image, one has to find, for each range block an appropriate mathematical transformation which when applied to an appropriate domain block gives rise to an approximation of the range block. This set of transformations thus obtained by partioning the whole image is called Partitioned Iterative Function System (PIFS).

Several researchers have suggested different algorithms with different motivations to obtain PIFS for a given image. We have already suggested a faster algorithm, to obtain PIFS, using Genetic Algorithms (GAs) [8, 9]. GAs [10, 11, 12] are optimization algorithms which are modeled according to the biological evolutionary processes. These optimization techniques reduce significantly the search space and time.

In the present work an attempt is made to use fractal codes as an input to an image magnification system. Some of the popular techniques of digital magnification of images are nearest neighbor, bilinear and bicubic interpolations. All these techniques are based on surface interpolation. In the interpolation techniques the global information are often ignored and only the local or semi global information are exploited. But in the proposed scheme the magnification task has been performed by using fractal codes where both the local and the global information are used. The scheme is nothing but a decoding scheme of fractal codes which gives rise to a magnified version of the original image. The article

reports the initial results of magnification using fractal codes which are obtained by a GA based technique [8, 9]. Comparison with the nearest neighbor image magnification methods has also been reported here.

In the magnification techniques the distortion due to blocking which is a local phenomenon is very usual. To quantify the amount of distortion, the widely used distortion measure is mean squared error (MSE) or some other version of it. MSE is a global measure which fails to account properly the local distortion due to blocking. But the blocking effects are very much sensitive to the human visual system. So, to quantify the global and the local distortions simultaneously a new distortion measure is introduced. The overall performances of the proposed algorithm is found to be satisfactory with respect to the new distortion measure.

Theory and key features of IFS and magnification using IFS are outlined in Section 2. The methodology of using fractal codes for magnification of a given image is described in Section 3. Section 4 presents implementation and the results. Discussion and conclusions are provided in Section 5.

2 Theory and Basic Principles

The detailed mathematical description of the IFS theory, Collage theorem and other relevant results are available in [3, 13, 14]. Only the salient features are discussed here. The theory of IFS and its use in image coding and in image magnification are described in the following subsections. The basic principle of Genetic Algorithms is also described.

2.1 Theoretical Foundation of IFS

Let I be a given image which belongs to the set X. Generally X is taken as the collection of compact sets. Our intention is to find a set $\mathcal{F}$ of affine contractive maps for which the given image I is an approximate fixed point. The fixed point

or attractor "A" of the set of maps $\mathcal{F}$ is defined as follows :

$$\lim_{N\to\infty} \mathcal{F}^N(J) = A, \quad \forall J \in X,$$

and $\quad \mathcal{F}(A) = A,$ where $\mathcal{F}^N(J)$ is defined as

$$\mathcal{F}^N(J) = \mathcal{F}(\ \mathcal{F}^{N-1}(J)\ ), \text{with}$$

$$\mathcal{F}^1(J) = \mathcal{F}(J), \quad \forall J \in X.$$

Also the set of maps $\mathcal{F}$ is defined as follows:

$$d(\mathcal{F}(J_1), \mathcal{F}(J_2)) \le s\ d(J_1, J_2); \ \forall J_1, J_2 \in X \quad \text{and} \quad 0 \le s < 1. \tag{1}$$

Here "d" is called the distance measure and "s" is called the contractivity factor of $\mathcal{F}$.

$$\text{Let } \ d(I, \mathcal{F}(I)) \le \epsilon \tag{2}$$

where ϵ is a small positive quantity. Now, by Collage theorem [1], it can be shown that

$$d(I, A) \le \frac{\epsilon}{1-s} \tag{3}$$

Here, $(X, \mathcal{F})$ is called iterative function system and $\mathcal{F}$ is called the set of fractal codes for the given image I.

2.1.1 Image Coding Using IFS

Let, I be a given image having size $w \times w$ and the range of gray level values be $[0, g]$. Thus the given image I is a subset of $\mathbb{R}^3$. The image is partitioned into n non overlapping squares of size, say $b \times b$, and let this partition be represented by $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_n\}$. Each $\mathcal{R}_i$ is named as range block where, $n = \frac{w}{b} \times \frac{w}{b}$. Let $\mathcal{D}$ be the collection of all possible blocks (within the image support) which is of size $2b \times 2b$ and let $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_m\}$. Each $\mathcal{D}_j$ is named as domain block with $m = (w - 2b) \times (w - 2b)$. Let,

$$\mathcal{F}_j = \{f : \mathcal{D}_j \to \mathbb{R}^3 \ ; \ f \text{ is an affine contractive map}\}.$$

Now, for a given range block $\mathcal{R}_i$, let, $f_{i|j} \in \mathcal{F}_j$ be such that

$$d(\mathcal{R}_i,\ f_{i|j}(\mathcal{D}_j)) \ \leq \ d(\mathcal{R}_i,\ f(\mathcal{D}_j)) \ \ \forall f \in \mathcal{F}_j,\ \forall j.$$

Now let k be such that

$$d(\mathcal{R}_i,\ f_{i|k}(\mathcal{D}_k)) \ = \ \min_j\{\ d(\mathcal{R}_i,\ f_{i|j}(\mathcal{D}_j))\ \} \tag{4}$$

Also, let $\quad f_{i|k}(\mathcal{D}_k) = \widehat{\mathcal{R}}_{i|k}$.

Our aim is to find $\widehat{\mathcal{R}}_{i|k}$ for each $i \in \{1, 2, \cdots, n\}$ or in other words for each range block ($\mathcal{R}_i$) we are to find appropriately matched domain block ($\mathcal{D}_k$) and appropriately matched map ($f_{i|k}$). Thus $\{\mathcal{D}_k, f_{i|k}\}$ is called fractal code for $\mathcal{R}_i$. Figure 1 illustrates the mapping of domain blocks to the range blocks.

2.2 Image Magnification Using Fractal Codes (IFS)

The transformation $f_{i|k}$ consists of two parts. The first part is contraction. This contraction operation is applied on the domain block in such a way that its size becomes equal to the size of the concerned range block. The second part is transformation. Here the transformations of rows and columns of contracted domain block to that of the range block are obtained by using eight possible isometric transformations [7] and simple coordinate geometry. The gray value transformation, in this part, is obtained by least square technique *i.e.* by fitting a straight line using two sets of pixel values of which one is from range block and other is from contracted domain block. Thus $f_{i|k}$ can be looked upon as mixture of two transformations, $f_{i|k} \ = \ t_{i|k}\mathcal{C}$, where, $\mathcal{C}$ is contraction operation and $t_{i|k}$ is transformation for rows, columns and gray values. As the size of the domain block is double that of the range block, an average of four neighboring pixels are taken to construct a contracted domain block for the implementation purpose of the contractive operator.

Thus we have, $I \ = \ \bigcup_{i=1}^{n} \mathcal{R}_i$ and using (2) we have,

$$d(\ \bigcup_{i=1}^{n}\mathcal{R}_i,\ \bigcup_{i=1}^{n}\widehat{\mathcal{R}}_{i|k}\ ) \ \leq \ \epsilon \tag{5}$$

Now, let $\mathcal{M}$ be a magnification operator such that

$$d(\bigcup_{i=1}^{n} \mathcal{R}_i, \bigcup_{i=1}^{n} \mathcal{M}(\mathcal{R}_i)) \leq \epsilon_1 \tag{6}$$

where ϵ_1 is a small positive quantity. Now by (5) and (6) we have,

$$d(\bigcup_{i=1}^{n} \mathcal{R}_i, \bigcup_{i=1}^{n} \mathcal{M}(\widehat{\mathcal{R}}_{i|k})) \leq \epsilon_2 \tag{7}$$

where ϵ_2 is a small positive quantity. Again, we have,

$$\widehat{\mathcal{R}}_{i|k} = f_{i|k}(\mathcal{D}_k) = t_{i|k}\mathcal{C} (\mathcal{D}_k).$$

So,

$$d(\bigcup_{i=1}^{n} \mathcal{R}_i, \bigcup_{i=1}^{n} \mathcal{M} \ t_{i|k} \ \mathcal{C}(\mathcal{D}_k)) \leq \epsilon_2. \tag{8}$$

Now, reconstruction of images using the operator $\mathcal{M}$ should be an inverse of contraction operation using the operator $\mathcal{C}$. So, by (7) and (8)

$$d \ (\ \mathcal{M} \ (\widehat{\mathcal{R}}_{i|k}) \ , \ t_{i|k}(\mathcal{D}_k) \ ) \leq \epsilon_3. \tag{9}$$

Hence

$$d \ (\ \bigcup_{i=1}^{n} \mathcal{R}_i \ , \ \bigcup_{i=1}^{n} t_{i|k}(\mathcal{D}_k) \ ) \leq \epsilon_4. \tag{10}$$

Both ϵ_3 and ϵ_4 are small positive quantities.

Thus from (10), it is clear that there is no need of constructing the magnification operator $\mathcal{M}$, only the second part of the fractal codes has to be applied on the domain block to get an image which is very close to the given image I and this image has size double that of the given image.

2.3 Genetic Algorithm

Genetic Algorithms (GAs) [10, 11, 12] are highly parallel and adaptive search and machine learning process based on a natural selection mechanism of biological genetic system. Parallelism of GAs depend upon the machine used for computations. GAs help to find the global optimal solution without getting stuck at local

optima as they deal with multiple points (called, chromosomes) simultaneously. To solve the optimization problem, GAs start with the chromosomal (structural) representation of a parameter set. The parameter set is coded as a string of finite length called chromosome or simply string. Usually, the chromosomes are strings of 0's and 1's. If the length of chromosome (string) is l then total number of chromosomes is 2^l. To find a near optimal solution, three basic genetic operators, i)Selection, ii)Crossover and iii)Mutation are exploited in GAs.

In selection procedure the objective function or the fitness function of each individual string is responsible for its selection as a new string in the next mating pool. We have used the elitist model of GAs where the worst string in the present generation is replaced by the best string of the previous generation. The crossover operation on a pair of strings is described here. An integer position k is selected randomly between 1 and $l-1$ ($l > 1$). Two new strings are then created by swapping all the characters from position $k+1$ to l of old strings. This process is very often in natural genetic system and thus, a high probability is assigned to indicate the occurrence of this operation. In mutation operation every bit of every string is replaced by the reverse character (i.e. 0 by 1 and 1 by 0) with some probability. Usually a low probability is assigned for mutation operation and the occurrence of this operation is guided by this probability. We have used the varying mutation probability scheme [15] to guide the mutation operation in the present work. Starting from the initial population (of strings) a new population is created using three genetic operators as described above. This entire process is called an iteration. In GAs a considerable number of iterations are performed to find the optimal solution. The string which possesses optimal fitness value among all the strings is called the optimal string. The optimality of the fitness value of strings is problem dependent. If the problem is a minimizing problem, the lowest fitness value is taken as the optimal one and the maximum fitness value is selected as the optimal one if the problem is a maximization problem. The convergence of GAs to an optimal solution is assured as the number of iterations increases [16].

The methodologies to obtain magnified images from fractal codes are now discussed below.

3 Methodology

So far we have discussed how to apply the fractal codes or IFS to get a magnified image which is double in size that of the given image. On successive applications of this proposed algorithm, magnification by factor 4, 8, 16 etc. can also be achieved. But the first task is to obtain the fractal codes or IFS for a given image.

3.1 Construction of IFS for magnification

The size of the range blocks plays an important role in image compression as well as magnification. If small blocks are taken, the finer details of the image are preserved and restored in the decompressed image but the compression ratio will be less. On the other hand more compression will be achieved, at the cost of finer image details, if large range blocks are considered. Thus a trade off has to be made to get quality decompressed image as well as considerable amount of compression. But the main task in magnification is only to restore all the image details and almost no emphasis is given on the amount of compression achieved. So, in this case, small range blocks are to be considered to keep track with every minute details of the original image.

In the proposed algorithm, to obtain the fractal codes of small range blocks of a given image, the blocks are first classified into two groups using a simple classification scheme [9]. The groups are formed according to the variability of the pixel values in the blocks. If the variability of a block is low *i.e.*, if the variance of the pixel values in the block is below a fixed value, called threshold, we call the block as smooth type range block. Otherwise we call it a rough type range block. The threshold value to separate the range blocks into two types is obtained from the valley in the histogram of the variances of pixel values of the blocks. All the pixel values in a smooth type range block are replaced by the mean of its pixel values. So, it is enough to store only the discretized mean value. On the other hand for each rough type range block, the appropriately matched domain block as well as appropriately matched transformation from

eight possible isometric transformations [7] have to be searched out. To solve this search problem a GA based technique [8, 9] is adopted. GA is a search technique which finds out the optimal solution faster than the exhaustive search technique.

3.2 GA to Find IFS

The parameters which are to be searched using GA are location (starting row and starting column) of domain block and its eight possible isometric transformations [7]. The realization of the first is two integer values between 1 and $w - 2b$ and the second can take any value between 1 and 8. Binary strings of length l are introduced to represent the parameter set. Here l is chosen in such a way that the set of 2^l binary strings exhausts the whole parametric space.

A string indicates the location and the isometric transformation of a domain block. In fractal codes we are to find an appropriate domain block and an appropriate transformation for each range block. In other words we are to find the appropriate string for each range block. Out of 2^l strings a few strings are selected randomly to start the GA. Starting with the initial mating pool and using the three basic operations new populations are generated in each iteration of the GA. After a large number of iterations, the GA will produce a near global optimal solution. To obtain the appropriate string in each step we are to calculate the fitness function of each string in each iteration. Mean square error (MSE) is used as fitness function of a string. In each mating pool, the strings first under go crossing over operation pairwise and the mutation operation is applied in each bit of each string.

In the next section the technique for successive magnification has been described.

3.3 Successive Magnification

In the case of successive applications of the algorithm, the fractal codes need not be computed afresh. The fractal codes that are used in a step are obtained

by modifying the fractal codes obtained in the previous step. In particular, the transformations $t_{i|k}$ are modified by using the image that is already obtained in the previous step. The locations of the appropriately matched domain blocks are kept fixed in all the steps. Only the size of the domain blocks is increased in the modification process of the fractal codes. Thus, in particular, only the gray level transformation in $t_{i|k}$ is to be modified. The gray level transformations are obtained using least square technique. In this technique a straight line is fitted with two sets of gray level values of which one is from range block and other is from contracted domain block. In the successive magnification scheme these two sets are enlarged. These enlarged sets are divided into several parts and separate straight lines are fitted for each parts using the same least square technique. For the magnification by a factor 4 the sets are divided into 4 parts and for the magnification by a factor 8, 16 parts are considered separately and so on. So, the number of fractal codes, in a step, becomes 4 times larger than its counter part in the previous step. So, it is enough to find the fractal codes in the case of magnification by a factor 2 and in other cases these codes are modified accordingly.The modified codes are then used for the magnification greater than two.

In the next section we have discussed the evaluation criteria to judge the performance of the proposed algorithm.

3.4 Fidelity Criteria

The magnified image has been reconstructed from the fractal codes which are obtained by using the GA based technique [8, 9]. The next task is to judge the performance of the proposed fractal based image magnification algorithm. For this purpose one has to measure the distortion between the given image and the reconstructed image. To quantify the amount of distortion, the widely used distortion measure is Mean Squared Error (MSE) or some modified form of it like Peak Signal to Noise Ratio (PSNR). MSE or PSNR examines the similarities between two images. But MSE is a size dependent measure *i.e.*, the two images, under consideration, should have the same size. Moreover it is a global measure

which is the average of pixel to pixel difference. It does not precisely indicate the large and significant local distortions due to blocking or blurring as it deals with the average distortions. But the blocking effects are very much sensitive to the human visual system. So, one has to think of a size independent measure which can keep track with local as well as global distortions simultaneously to judge the performances of magnification task. A new fidelity criteria whose performance is also similar to that of visual judgment is introduced to find out the distortion between the given image and the magnified reconstructed image. The overall performances of the proposed algorithm is found to be satisfactory both in the light of this new distortion measure (objectively) as well as from visual judgment (subjectively).

The images that are obtained from the codes usually have specific artifacts such as blocking, ringing and blurring. Actually these artifacts are reflected more prominently in the high frequency component of the image and are very sensible to the human visual system [17]. So, in our proposed error measure, we have tried to measure the errors in edges. For simplicity only the vertical and horizontal edges have been considered.

Both the images are first partitioned into blocks proportional to their respective sizes in such a way that both images contain equal number of blocks. The error is then measured block wise and finally the average error is noted. To detect the edges of each block we have used the scheme suggested by Ramamurthi et al. [18]. The edge blocks consist of value "0" and "1" where, "1" represents the presence of edge. Now it is expected that the original and the magnified blocks should have same type of edge distributions. In other words the expected run of "1" present in both the blocks should be same if normalized by their respective sizes. Thus the error measure is defined by the difference between the normalized expected "run" of "1" present in the given image and in the magnified reconstructed image. The vertical and horizontal edges are considered separately and then averaged to give rise the final error measure of a block. The algorithm of the proposed error criteria is described below.

3.4.1 Description of the algorithm

Step 1 : Partition the images, I_1 and I_2 (with size of I_1 less than size of I_2) into square blocks such that the number of partitions is same in both the images. Let p_1 and p_2 (with $p_1 < p_2$) be the sizes of the square blocks for the images I_1 and I_2 respectively. Let these blocks be $B_{11}, B_{12}, \cdots, B_{1n}$ and $B_{21}, B_{22}, \cdots, B_{2n}$.

Step 2 : From $B_{ij}, i = 1,2$ and $j = 1,2,\cdots,n$ compute gradient matrices. Let G_{ij}^h and G_{ij}^v be respectively the horizontal and the vertical gradient matrix. The elements of the gradient matrices are all either 0 or 1. The gradient matrices are defined as follows

$$
\begin{aligned}
G_{ij}^h(m,n) &= 0 \;\; if \;\; \frac{|g_{m,n} - g_{m,n+1}|}{\frac{g_{m,n}+g_{m,n+1}}{2}} < T \\
&= 1 \;\; if \;\; Otherwise.
\end{aligned}
$$

and

$$
\begin{aligned}
G_{ij}^v(m,n) &= 0 \;\; if \;\; \frac{|g_{m,n} - g_{m+1,n}|}{\frac{g_{m,n}+g_{m+1,n}}{2}} < T \\
&= 1 \;\; if \;\; Otherwise.
\end{aligned}
$$

Here $g_{m,n}$ = Gray level value of $(m,n)th$ pixel in a block and T = A prefixed threshold value.

Step 3 : Find the expected run of 1 present in both horizontal and vertical directions in both G_{ij}^h and G_{ij}^v. Let L be the random variable denoting the number of run of 1 in a particular gradient matrix in a particular direction. Compute $E_{ij}^{hh}(L)$, $E_{ij}^{hv}(L)$, $E_{ij}^{vh}(L)$, $E_{ij}^{vv}(L)$. Here expected run of 1 is defined as

$$E(L) = \sum_k k \frac{Number\ of\ times\ the\ run\ of\ length\ k\ appears}{Total\ number\ of\ run\ (of\ all\ possible\ lengths)\ present}.$$

Now compute

$$E_{ij}(L) = \frac{E_{ij}^{hh}(L) + E_{ij}^{hv}(L) + E_{ij}^{vh}(L) + E_{ij}^{vv}(L)}{4}.$$

Step 4 : Normalize $E_{ij}(L)$ by respective block size.

$$
\begin{aligned}
E_{ij}(L) &= \frac{E_{ij}(L)}{p_1 \times p_1} \;\; if \;\; i = 1 \\
&= \frac{E_{ij}(L)}{p_2 \times p_2} \;\; if \;\; i = 2
\end{aligned}
$$

Step 5 : compute the final error measure E between the two given images I_1 and I_2. E is defined as

$$E = \frac{1}{n} \sum_{j=1}^{n} \{E_{1j}(L) - E_{2j}(L)\}^2.$$

The next section deals with the implementation and the experimental results of the proposed algorithm.

4 Implementation

To find the fractal codes for a given image the search is to be made for all possible domain blocks as well as eight possible isometric transformations [7]. To reduce the search space and time Genetic Algorithm is used instead of exhaustive search. The search space reduction is achieved since near optimal solutions are usually satisfactory and, intuitively, solutions which are far away from the expected are rejected in a probabilistic manner. This is the reason for GA to perform well for optimization problems. We have already shown that the performances of GAs to find fractal codes of a given image [9]. The results are quite satisfactory and at least 20 times reductions in the search spaces are achieved.

For the specific implementation of the proposed algorithm a part of the original "Lena" image (Figure 2(a)) is treated as the original input image. The input image is a 128×128, 8 bit/pixel image. The GA based technique [9] is applied to generate the fractal codes. Moreover a simple classification scheme [9] for range blocks have been adopted to retain the details of similar looking regions of the given image more easily.In classification scheme, the range blocks are grouped into two classes *viz.*, "smooth" and "rough". In the case of magnification algorithm, small range blocks of size 2×2 are considered for the computation of fractal codes. Using these codes an image of size 256×256 is reconstructed which is two times magnified than the original image. This image is found to be very close to the original image which is judged by the error measure as described in section 3.4. These fractal codes are then modified stepwise to get the images which are 4 times and 8 times magnified than the original one. In each step the

error, in comparison to its previous step, is measured successively. The proposed algorithm is also compared in terms of new distortion measure with the nearest neighbour technique for image magnification. Nearest neighbour is the simplest method of digital magnification. Given an image of size $w \times w$, to magnify it by a factor k, every pixel in the new image is assigned the gray value of the pixel in the original image which is nearest to it. This is equivalent to repeating the gray values $k \times k$ times to obtain the magnified image. The resultant image for large magnification factors will have prominent block like structures due to lack of smoothness.

The proposed algorithm has also been implemented on a "Low Flying Aircraft" (LFA) image having size 128×128 and range of gray level values 0 to 255. Other parameters of the algorithm are kept fixed as in the case of "Lena" image. All the results obtained are presented in Table 1. The original and magnified images of "Lena" and "LFA" images are shown in Figure 2 to Figure 7.

Table 1: Test results obtained in Image magnification Algorithms

Image	Reconstructed Image Statistics								
	MF	Error		MF	Error		MF	Error	
		Fractal	NN		Fractal	NN		Fractal	NN
Lena	2	1.18	1.62	4	1.37	3.11	8	1.24	6.15
LFA	2	2.32	2.37	4	2.85	4.59	8	2.43	9.11

MF=Magnification Factor and NN=Nearest Neighbour

5 Discussion and Conclusions

An important advantage of fractal image magnification is that it magnifies the image by expanding the fractal codes or the transformations which may be looked upon as independent of image resolution. The only error involved with it is the problem of discretization. Thus the structure and the shape of the image remains almost same. In a sense, it is like interpolation resulting in a sharper expanded

image. Other image magnification schemes use pixel replication to expand image. Pixel replication makes an image blocky, blurry and patchy after a certain extent of expansion.

The proposed technique of fractal image magnification utilizes the coded (fractal) version of the input image instead of the original image. Therefore it is cost effective in the sense of storage space and time as no decoding is performed at the receiving end in case of transmission of the codes.

In this article fractal image magnification algorithm is implemented with magnification factors which are multiples of 2. But in practice one may need to magnify the given image by other factors too. One way of performing this task is to select the domain blocks which are 3 times larger than that of the range block. Now the image can be magnified 3 times using the corresponding fractal codes. Hence magnification by factors which are multiples of 3 can be achieved. On the other hand, magnification by factors which are not multiples of 2 or 3 can also be achieved by considering the normalized distance between the range block and its matched domain block in the original image. While performing the magnification task, if the distance between the range block and the domain block appears to be fractional, one has to discretize it properly. Moreover this normalized distance can also be utilized for the magnification by any factor.

The size of the range blocks considered plays a vital role in fractal image compression and fractal image magnification (ref. section 3.1). In particular these two algorithms are in opposite direction from the point of view of range block size. So, one can think of an optimal range block size for which good quality magnified images can be reconstructed from the fractal codes and at the same time considerable amount of compression (in terms of compression ratio) can be achieved. To solve this problem one can think of quadtree partitioning of the images instead of square partitioning while generating the fractal codes [19]. Another scheme, to obtain the IFS codes of an image, suggested by Thomas et al. [20] can also be adopted in this connection. In this scheme they have considered irregular shaped range blocks. Automatically the matched domain blocks are just the scaled and magnified version of these irregular shaped range blocks.

In the present article we have introduced a new distortion measure or fidelity criteria to judge the performance of the proposed algorithm. Instead of using this distortion measure one can adopt some nonparametric statistical tests [21] for the same purpose. The common tests for examining the degree of association between two distributions whose distribution functions are unknown are Sign test, Wald Wolfowitz Run test, Wilcoxson test and Kolmogorov Smirnov test. Another evaluating criteria based on fractal dimension has already been suggested by Lalitha et al. [22]. But the most important feature which should be considered while examining the similarities between two images is the edge distribution of the images as the edges are very sensitive to human eyes. But neither the fractal based evaluating criteria nor the statistical tests take care of distortions present in the edges. So, one of the important tasks is to find the proper edges in the images for the implementation of the proposed distortion measure. We have used a very simple technique for the detection of edges though one may suggest more complex techniques for it. The overall performance of the proposed fractal based image magnification technique is good. The proposed distortion measure also appears to be satisfactory and compatible with visual judgment.

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Figure Captions

Figure 1 : Mapping from Domain blocks to Range blocks.

Figure 2(a) : Original "Lena" image.

Figure 2(b) : Decoded "Lena" image.

Figure 3(a) : Two times magnified image of "Lena" using Fractal Image Magnification Technique.

Figure 3(b) : Four times magnified image of "Lena" using Fractal Image Magnification Technique.

Figure 3(c) : Eight times magnified image of "Lena" using Fractal Image Magnification Technique.

Figure 4(a) : Two times magnified image of "Lena" using Nearest Neighbour Technique.

Figure 4(b) : Four times magnified image of "Lena" using Nearest Neighbour Technique.

Figure 4(c) : Eight times magnified image of "Lena" using Nearest Neighbour Technique.

Figure 5(a) : Original "LFA" image.

Figure 5(b) : Decoded "LFA" image.

Figure 6(a) : Two times magnified image of "LFA" using Fractal Image Magnification Technique.

Figure 6(b) : Four times magnified image of "LFA" using Fractal Image Magnification Technique.

Figure 6(c) : Eight times magnified image of "LFA" using Fractal Image Magnification Technique.

Figure 7 : Eight times magnified image of "LFA" using Nearest Neighbour Technique.

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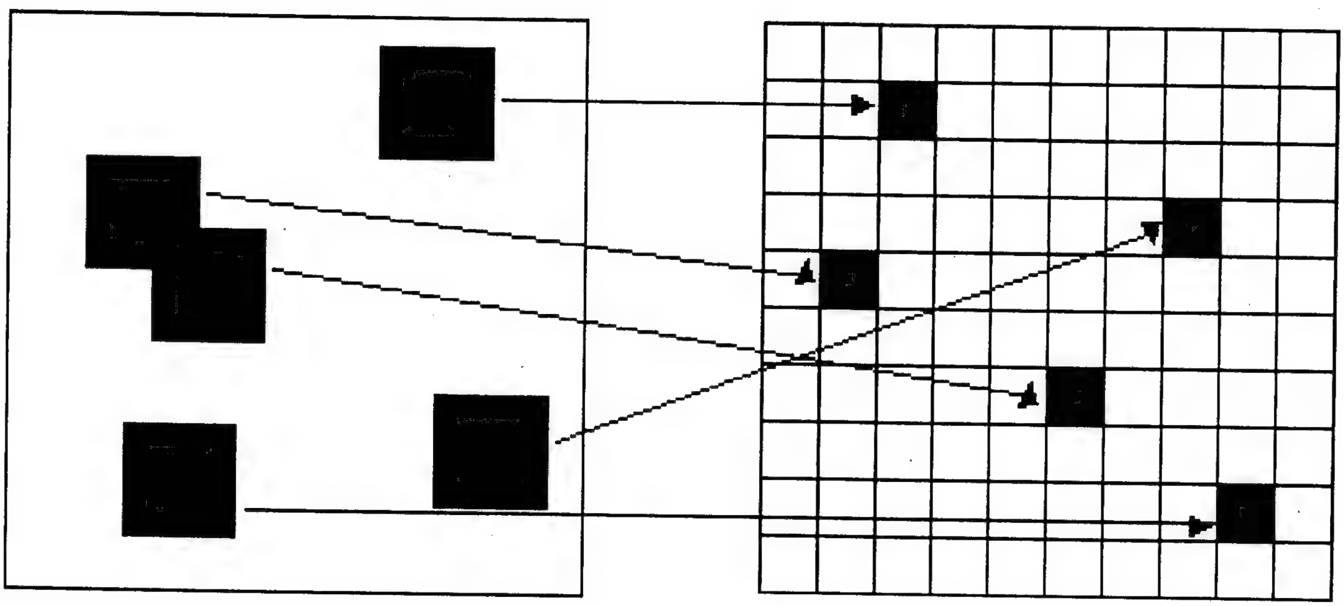

Fig. 1

Fig. 2 (a)

Fig. 2 (b)

Fig. 3 (a)

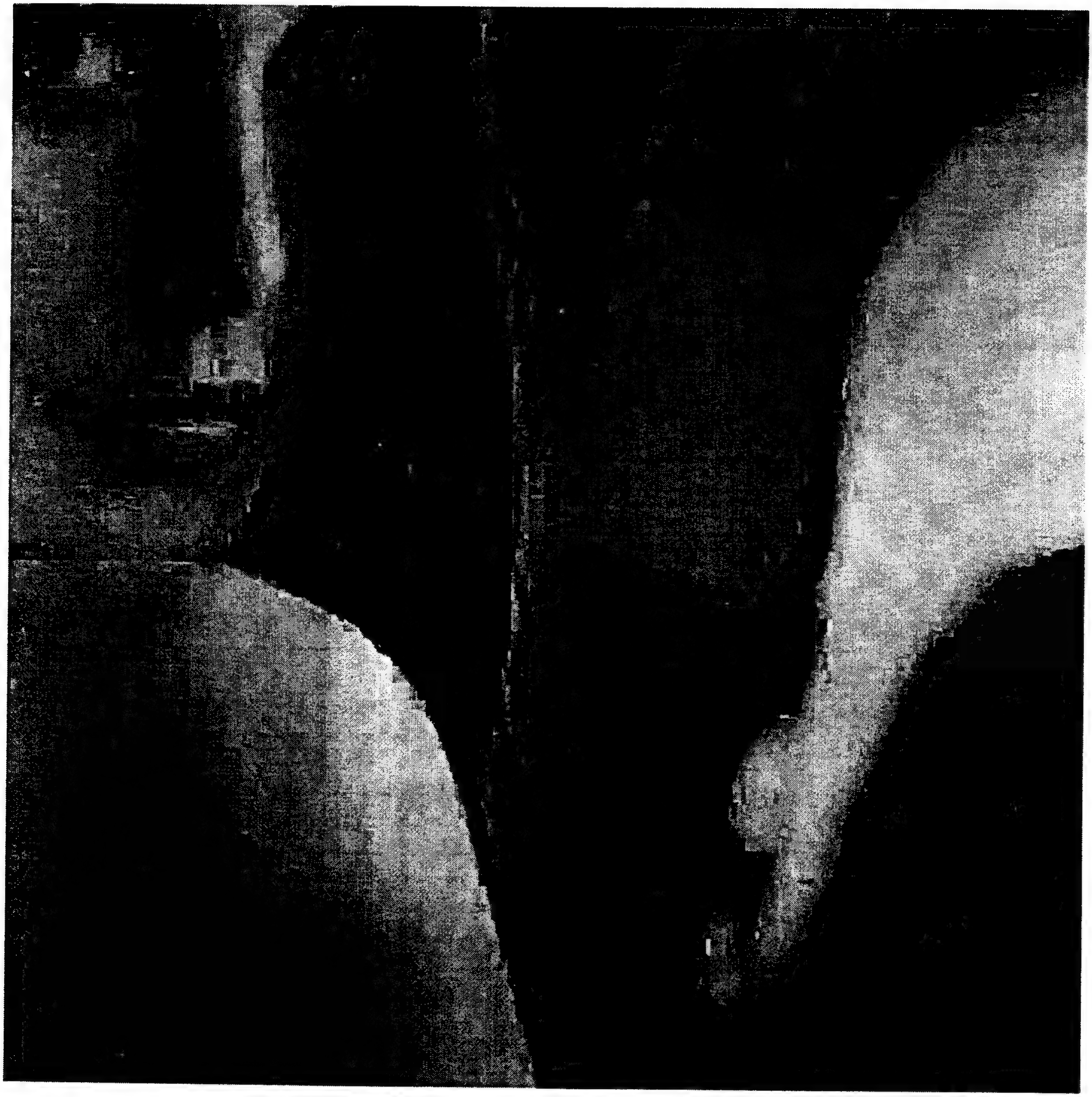

Fig. 2b

Fig. 4(a)

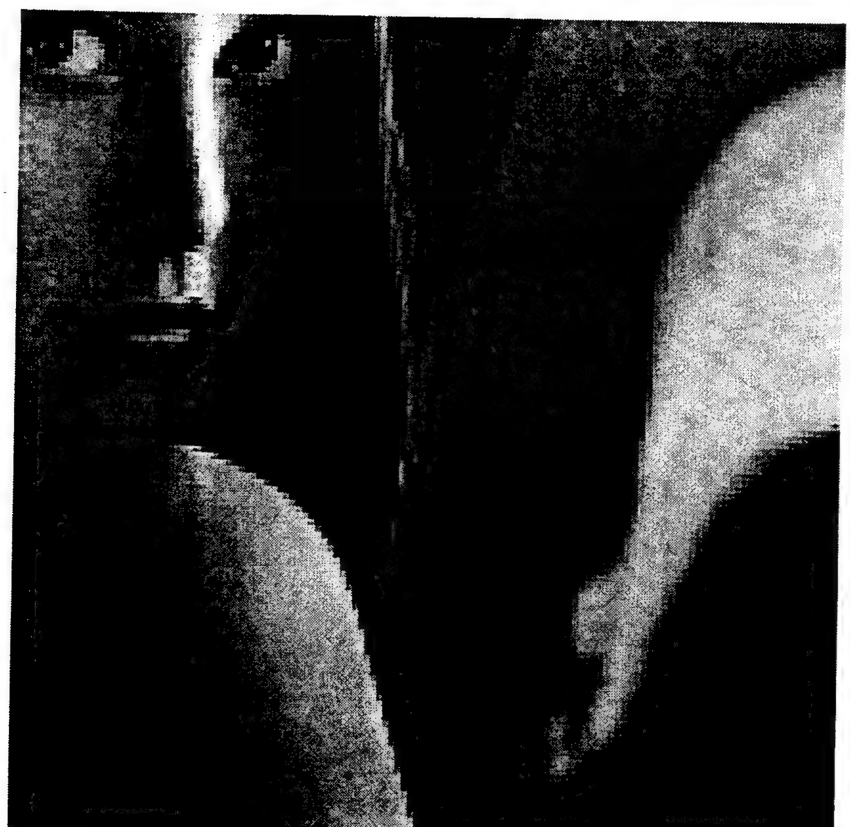

Fig. 4(b)

Fig. 1(e)

Fig. 5(a)

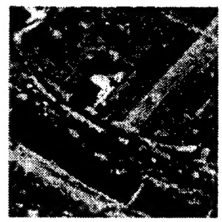

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Fig. [illegible]

Fig 6(a)

Fig. 7.